Traffic flow models with phase transitions: theory and numerical approximations

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Abstract

We consider a traffic flow model with phase transitions, that has been introduced by R.M. Colombo in [4]. For low densities, the flow is *free* and is described by a scalar conservation law usually referred to as the Lighthill-Whitham [8] and Richards [9] model. At high densities, the flow is *congested* and is described by the 2×2 system introduced in [3]. The coupling is achieved by introducing a transition dynamics between free and congested flow.
Other traffic flow models with phase transitions have been considered in the literature since the 60-ties, in order to explain empirical flow-density relations. In particular, we recall the scalar model of Drake, Schofer and May [6]. Another model has been introduced recently the author [7]. It consists in coupling the LWR equation with the 2×2 Aw-Rascle model [1].

In [5], we prove the well posedness (in the L^1-norm) of the solutions to the Cauchy and initial-boundary value problems for the model under consideration, for data with bounded total variation. More precisely, we construct a *Riemann Semigroup* of solutions defined on a set of functions with bounded total variation, which is Lipschitz continuous with respect to initial (and eventually boundary) data and time. From the traffic point of view, the result is useful in view of applications to control and optimization problems.

As far as numerical approximations are regarded, we observe that, due to the lack of convexity of the domain, the classical Godunov method does not apply. In fact, in the presence of phase transitions, the projection step of the algorithm can give values which are not in the domain.
In [2], we present a new version of the Godunov scheme, based on a modified averaging strategy and a sampling procedure. More precisely, we modify the mesh cells following the phase boundaries, so that the projection involves only values belonging to the same phase. In order to come back to the original cells, we complete the projection step with a Glimm-type sampling technique.

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