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Lattice Boltzmann 2038

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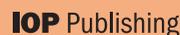
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Lattice Boltzmann 2038

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Abstract – Based on the past twenty-five years of lattice Boltzmann research, we venture into a far-flung prediction for the next twenty-five, with past and future privileged over the present state of affairs.



perspective

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Introduction. – The Lattice Boltzmann (LB) method made its earliest chronological appearance in 1988 [1], its first computationally viable realization being published by *EPL* shortly later [2]. Hence LB celebrates its 27th birthday this year: over a quarter of century which has witnessed a burgeoning growth, as reflected by its ponderous bibliographic record: over 10^5 ISI citations with $h > 130$, as of this writing. The question naturally arises: how about the next near twenty-five, how about Lattice Boltzmann 2038? In this Perspective, I shall engage in a rather visionary, and yet hopefully not reckless, projection of the LB method some twenty-five years down the line.

The LB trailblazers. – LB was born in the wake of the disenchantment with the grand promise raised by Lattice Gas (LG), the lattice Boolean analogue of the Newton equations for fluids [3]. LG raised enormous hopes as a new paradigm for the numerical simulation of fluid flows, most on account of its revolutionary nature: exact Boolean calculus for fluids. At a closer scrutiny, however, LG did not live up to the (huge) expectations for a number or reasons, primarily statistical noise and the computational complexity of the collision rule, which scales exponentially with the number of bits attached to each lattice site. One could say that LG just suffered the fact that what we call general purpose computers are in fact special purpose computers for floating point operations! Be as it may, LG did not make into a major mainstream in computational fluid dynamics. The stepping stone of the earliest Lattice Boltzmann scheme [1] was the practical realization that pre-averaging the noisy LG configuration, via a Boltzmann formulation, is way more

efficient than post-averaging the Boolean LG dynamics. In this respect, LB is appropriately viewed as a pre-averaged LG [4]. This solved the noise problem, but left all other issues untouched, including the aforementioned exponential complexity barrier. However, this triggered a series of developments which led to the solution of all major shortcomings of LG in a short sequence of papers between 1989 and 1992, many of which happen to have been published in *EPL* [5–10]. Among others, a very influential outcome for most subsequent developments was the so-called *top-down* strategy, namely designing the LB equations from the symmetry requirements of the macroscopic target (Navier-Stokes), instead of deriving them from an underlying microscopic theory [6]. Compliance with the second principle (*H*-theorem) went lost in the process, but brilliantly recovered a few years later through the development of the so-called entropic LB method (ELB) [11].

So much for the basic LB “technology”, as we know it today. Let us now proceed to discuss the main ingredients which make it work.

The four magic ingredients. – The LB equation (LBE) reads as follows:

$$f_i(\vec{x} + \vec{c}_i, t + 1) - f_i(\vec{x}, t) = -\Omega_{ij}(f_j - f_j^{eq})(\vec{x}; t) + S_i(\vec{x}; t), \quad (1)$$

where the lattice time step is made unit for simplicity. In the above, $f_i(\vec{x}, t) \equiv f(\vec{x}, \vec{v} = \vec{c}_i; t)$, $i = 0, b$, denotes the probability of finding a particle at lattice site \vec{x} and time t with a molecular velocity $\vec{v} = \vec{c}_i$. Here, \vec{c}_i denotes a set of discrete velocities, which must exhibit enough symmetry to obey the mass-momentum (energy) conservation rules, along with rotational invariance (isotropy).

Typical two- and three-dimensional lattices are shown in fig. 1.

The left-hand side of eq. (1) denotes free-streaming, while the right-hand side is the interaction part, which consists of a short-range component, driving the system towards a local equilibrium f_i^{eq} and a soft-core source of momentum $S_i(\vec{x}; t)$. The latter is in fact more general and can represent the source of any quantity of relevance to the physics in point.

The local equilibrium corresponds to a finite-order truncation of a local Maxwellian:

$$f_i^{eq} = w_i \rho \left(1 + \beta u_i + \frac{\beta^2}{2} q_i \right), \quad (2)$$

where $\beta = 1/c_s^2$, $u_i = \vec{u} \cdot \vec{c}_i$ and $q_i = u_i^2 - u^2 c_s^2$, c_s and \vec{u} being the lattice sound speed and the local flow velocity, respectively. In the above w_i is a set of weights normalized to unity, the discrete analogue of the absolute Maxwellian in continuum velocity space.

The truncation is an unavoidable consequence of lattice discreteness, which permits to recover Galilean invariance only to a finite order in the Mach number $Ma = |\vec{u}|/c_s$.

The relevant hydrodynamic quantities are computed as simple linear combinations of the discrete distributions, namely

$$\rho(\vec{x}; t) = \sum_i f_i(\vec{x}; t), \quad \rho \vec{u}(\vec{x}; t) = \sum_i \vec{c}_i f_i(\vec{x}; t), \quad (3)$$

where ρ is the fluid density. Higher-order moments deliver the fluid pressure and stress tensor in the form of *linear and local* combinations of the discrete populations, which proves very convenient for simulation purposes.

Formally, The LB equation (1) is nothing but a set of finite-difference equations, and yet one with great power inside.

This power stems mainly from four basic ingredients, namely: *i) Exact free-streaming*, *ii) local lattice equilibria*, *iii) tunable relaxation matrix*, *iv) flexible external source*. Before moving on to these items, we hasten to add that eq. (1) can be shown to converge to the (quasi-incompressible) Navier-Stokes equations in the usual limit of small Knudsen numbers, $Kn = \lambda/L \ll 1$, *i.e.* small mean free path *vs.* the typical scale of variation of hydrodynamic quantities. This is also a statement of weak departure from local equilibrium.

Technically, this entails a Taylor expansion in the lattice time step, as combined with a double expansion in low Knudsen and Mach numbers. The tool of the trade is the Chapman-Enskog asymptotics [12].

Free-streaming. The left-hand side of eq. (1) is the lattice analogue of the free-streaming operator $\vec{v} \cdot \nabla f$ of the continuum Boltzmann equation. Since $\vec{v} = \vec{c}_i$, the lattice streaming proceeds along straight lines (lightcones $d\vec{x}_i = \vec{c}_i dt$), a simple forward time-marching transfers the populations f_i from site to site with *zero loss* of information: the lattice streaming is *exact!* This is perhaps

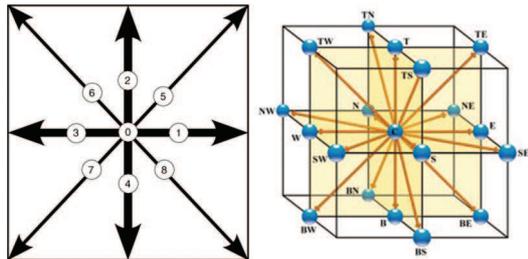


Fig. 1: (Colour on-line) The workhorse lattices in two and three dimensions, with 9 and 19 discrete velocities, respectively.

the most crucial asset of the LB method, a gracious inheritance of the LG dynamics. The fact that information always travels along straight lines, no matter how complex the fluid configuration, proves key also in connection with the handling of complex geometrical boundaries. To appreciate the point, one should contrast with the analogue convective term in the Navier-Stokes equations, namely $\vec{A} \equiv \vec{u} \cdot \nabla \vec{u}$, which shows that the fluid momentum is transported along material lines $d\vec{x}_u = \vec{u}(\vec{x}; t) dt$, way more complicated trajectories in the case of, say, turbulent flows.

Local equilibria. Another bonus of the LB formulation is that non-linearity and non-locality are disentangled; the non-local streaming is linear towards the discrete distribution f_i , and the non-linear collision operator is entirely local in space and time. This contrasts again with the fluid-dynamic representation, whereby the advection term \vec{A} is non-linear and non-local at a time. Moreover, local equilibria encode mass-momentum energy conservation to machine accuracy, which is a highly desirable feature for numerical implementation.

Tunable relaxation matrix. Once a proper set of discrete eigenvalues A_i^k , $i, k = 0, b$ is available, the collision matrix can be expressed in spectral form [13,14] $\Omega_{ij} = \sum_{k=0}^b A_i^k \omega_k A_j^k$, where ω_k are the tunable eigenvalues, *i.e.* the inverse relaxation time of the k -th order kinetic moment $M_k(\vec{x}; t) = \sum_{i=0}^b f_i(\vec{x}; t) A_i^k$. The lowest-order kinetic eigenvectors are readily imported from the corresponding Hermite polynomials, $A_i^0 = 1_i$, $A_i^{1,2,3} = c_{ix}, c_{iy}, c_{iz}, \dots$. The eigenvalues associated with mass and momentum are chosen to be zero in order to secure conservation. The leading non-zero eigenvalue, associated with the momentum-flux eigenvector, $c_{ia} c_{ib} - c_s^2 \delta_{ab}$, permits to control the viscosity according to the relation

$$\nu = c_s^2 (1/\omega - \Delta t/2), \quad (4)$$

where Δt is the time-step. Thus, by choosing $\omega \Delta t = 2(1 - \epsilon)$, it is possible to realize a fluid viscosity ϵ -times smaller than the continuum viscosity $\nu_c = c_s^2 \tau$, $\tau \equiv 1/\omega$ being the mean collision time. This property is crucial to access very low viscosity regimes which characterize fluid turbulence. Key to this property is the negative term in eq. (4), often called propagation viscosity, since it

derives from second-order Taylor expansion of the discrete streaming operator. To be noted is that this feature is inherent in LB, and not shared by the continuum Boltzmann equation. In other words, interaction with the lattice results in a negative viscosity which subtracts to the collisional one.

Source term. The external source describes the coupling of the discrete fluid to the external environment, typically in the form of exchanges of mass/momentum/energy. Particularly relevant to LB theory is the case of an external force responsible for streaming in velocity space, namely: $S_v \equiv \frac{\vec{F}(\vec{x};t)}{m} \cdot \nabla_v f(\vec{x}, \vec{v}; t)$. In order to preserve the exact nature of the free-streaming operator, it proves expedient to move the term S_v to the right-hand side and treat it as a soft-core collision term. Such term injects $\int S_v m \vec{v} d\vec{v} = \rho \vec{F}$ units of momentum per unit time, so that, to the lowest order, the lattice source term simply reads

$$S_i \sim w_i \rho \vec{c}_i \cdot \vec{F}.$$

This simple expression illustrates a very powerful strategy to include the effects of external forces and potential energy interactions (in actual practice more sophisticated expressions are needed [15], but the concept stays the same).

Dark side of the LB moon. – Not all what glitters is gold, and LB has its own inconveniences too.

First, by working on a uniform spacetime crystal, LB is not well suited to body-fitted coordinates and adaptive time-stepping. This may constitute an obstacle for a certain type of engineering flows [16].

Second, being a natural-born dynamic scheme, LB is not a method of choice for steady-state computations.

Third, since lattice equilibria are truncated Maxwellians, they are not positive-definite above a given Mach number, typically of the order of 0.1, which means that LB is most suited to quasi-incompressible flows. Finally, while it is true that the implementation of complex boundary conditions is conceptually straightforward, the actual coding can be laborious. Most of the above can be remedied by advanced formulations of the method [17], but not without some toll of complexity. This said, it seems fair to state that the upsides of LB largely overshadow its weaknesses, whence the blooming of applications, which we are going to sketch in the sequel.

LB hydrodynamics. – As of today, LB has found applications across virtually all walks of fluid dynamics, from large-scale turbulence, to low-Reynolds flows in porous media, all the way down to biopolymer translocation [18]. The main assets, strictly stemming from the four ingredients discussed before, are high amenability to parallel computing, flexibility towards complex geometries and enhanced stability in low-viscosity regimes [19,20].

Thermal flows remain somehow open-ended; even though several thermal LB options are available to date [21–23], they still “suffer” from the inherent athermal nature of LB, a set of zero-temperature discrete beams in

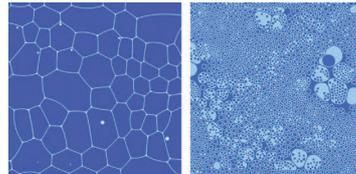


Fig. 2: (Colour on-line) Dry (left) and wet (right) foamy fluids obtained by a two-component Shan-Chen model. These foamy fluids show a very rich relaxation scenario, still the object of much debate in soft matter and the rheology of complex fluids. Courtesy of M. Sbragaglia.

velocity space. Therefore, some researchers still prefer to handle heat transport via other grid methods, leaving LB only for the mass-momentum transport equations.

Non-ideal fluids and multiphase flows. A major area of LB application is the simulation of a variety of multiphase and multicomponent flows [24–26]. Here, the main asset is the flexibility of the source term and/or local equilibria towards the inclusion of non-ideal interactions. A particularly popular expression is the one proposed by Shan and Chen,

$$\vec{F}(\vec{x}) = \Psi(\vec{x}) \sum_{i=0}^b G(\vec{x}, \vec{x} + \vec{c}_i) \vec{c}_i \Psi(\vec{x} + \vec{c}_i),$$

where $\Psi(\vec{x})$ is a local functional of the fluid density $\rho(\vec{x})$. By proper choice of this functional, the main features of non-ideal fluids, namely a non-monotonic equation of state supporting phase transitions and non-zero surface tension can be incorporated at a minimum programming effort. This simple variant opens up a vast scenario of applications involving multiphase and multicomponent flows, including foams and emulsions (see fig. 2).

Needless to say, this variant comes with a number of limitations, such as spurious interface currents, which severely constrain the accessible range of density ratios between the liquid and vapor phase. Yet, owing to its simplicity and efficiency, the method has gained increasing popularity over the years. Subsequent developments have improved significantly over the original version, but much remains to be done to gain further accuracy, especially in terms of multigrid/multiscale procedures at complex fluid interfaces. Another important issue is the incorporation of finite-temperature fluctuations for nanoscale flows.

Beyond Navier-Stokes. – As mentioned in the introduction, the LB formulation enjoys a solid asymptotic back-up in the hydrodynamic regime of low Knudsen numbers. Outside such regime, theoretical back-up becomes shaky. This is why, at some point LB was categorically banned out as totally unviable for any application beyond strict hydrodynamics.

Fortunately, such categoric no-go’s have been rapidly obsoleted by further developments of the method. This is *not* to say that LB can match Boltzmann or, less so,

molecular dynamics. The simple fact is that there exist a number non-equilibrium flows which do not fit the continuum Navier-Stokes description and yet do not require the inclusion of molecular details either. For those flows, molecular dynamics is simply an overkill. We shall refer to this situation as *Weakly Broken Universality* (WBU) as opposed to the unbroken universality of the Navier-Stokes equations. The last decade has witnessed major progress of LB methods towards the WBU region and consequent application to many flows of direct relevance to soft matter research. The key ingredients to this major leap are as follows: *i) High-order lattices, ii) kinetic boundary conditions, iii) proper tuning of the kinetic modes.*

High-order lattices. We have seen before that $b + 1$ discrete distributions f_i map one-to-one into an equal number of kinetic moments M_k . The kinetic moments relevant to (isothermal) hydrodynamics are density, momentum and momentum flux, *i.e.* moments up to second order. To describe large departures from Maxwell equilibria, hence strong non-equilibrium effects, the correct recovery of higher-order moments is imperative. This necessarily entails the use of Higher-Order Lattices (HOL) with several tens of discrete velocities [27]. Typical 33 and 93 speed lattices are shown in fig. 3. Such HOLs permit to capture non-trivial beyond-hydrodynamic phenomena, such as slip flow in microchannels, but clearly add to the complexity of the scheme, especially in conjunction with solid boundaries.

How far can one take LB in the non-equilibrium territory? This stands as one of the most interesting questions in current LB research, with potentially far-reaching implications for future multiscale down-coupling to the “true” Boltzmann or molecular dynamics.

Kinetic boundary conditions. The use of HOL may remain ineffective in the absence of proper boundary conditions. This is only natural, since typical soft-matter micro- and nanoflows are characterized by very high surface/volume ratios, in which the interaction of the fluid with confining walls plays a defining role. The essence of kinetic boundary conditions is to specify this interaction via mesoscopic accommodation coefficients, like in rarefied-gas dynamics [28]. As the nanoscale is approached, accommodation coefficients may eventually give way to suitable fluid-wall lattice potentials. With these ingredients in place, LB has proven capable of filling a number of notable gaps between Navier-Stokes and Boltzmann or Navier-Stokes and molecular dynamics. Most remarkable cases in point are microflows and droplet motion over corrugated surfaces, micro-nano emulsions, multiphase flows with suspended particles [29–32].

Tuning of kinetic modes. As discussed in the early part of this paper, the streaming step of the Boltzmann dynamics acts as a source of non-equilibrium, which is then pulled back to equilibrium by the effect of collisions. In several applications, it proves convenient to “regularize” the LB dynamics by projecting out the kinetic modes after

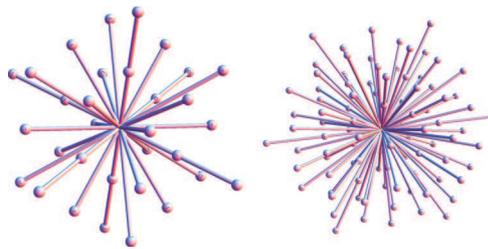


Fig. 3: (Colour on-line) Three-dimensional higher-order lattices, with 33 and 93 speeds, respectively. By extending beyond the first Brillouin cell, these lattices permit to achieve higher-order isotropy of the relevant LB tensors and capture (some) non-equilibrium effects carried by higher-order kinetic moments. In addition, they also permit to define more general lattice potentials, with coexisting/competing attractive and repulsive interactions. Courtesy of A. Montessori (see also [33]).

the streaming step. This proves beneficial to the stability of hydrodynamic flows and also to achieve quantitative accuracy in the non-hydrodynamic regime [34,35].

Heading towards soft-matter flows. – Another major direction of special (but not exclusive) relevance to soft-matter applications is the extension of LB techniques to the case of flows with suspended bodies. This is a fast-growing area of LB research, with applications across virtually all allied fields of fluid mechanics, such as chemical engineering, material science, biology and medicine. Following the pioneering work of Ladd, which first implemented the case of spherical colloids within a *fluctuating* LB [36], many variants have become available today.

Among others, perhaps most promising is the merger between LB for the fluid solvent and the Immersed Boundary Method (IBM) for the suspended object [37]. The IBM is a very elegant and powerful method to deal with motion of rigid or deformable moving bodies within the flow. The basic idea is to treat the fluid (solvent) in Eulerian form, hence well suited to LB, and the immersed body as an off-grid Lagrangian structure (membrane). The membrane exerts a boundary force on the fluid, whose motion, in turn, determines the velocity on the membrane. The force exerted by the membrane at the fluid location \vec{x} at time t is expressed by a double integral over the entire surface of the force \vec{F} , emanating from each single point $\vec{s} = (\xi, \eta)$ of the membrane. On the assumption that each point of the membrane moves at the same speed as the fluid, the equation of motion of the membrane is dynamically coupled to the fluid equations, using a Lagrangian particle method for the former and an Eulerian one (LB in our case) for the latter. This set of simultaneous equations is generally solved by iteration, until the condition of zero relative motion between the membrane and the fluid is fulfilled. In fact, such a condition is most conveniently regarded as a Lagrangian constraint on the fluid equations.

These techniques have permitted to model many sorts of colloidal flows, including blood flow in human arteries,

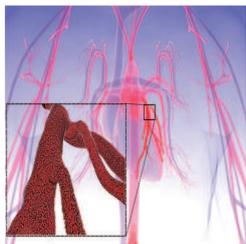


Fig. 4: (Colour on-line) Multiscale LB simulation of blood flow in a human artery. The arterial geometry is taken from real-patient data and converted, after proper smoothing, into a digital grid. The inset shows the granular structure of the blood flow at the scale of single red cells. Courtesy of S. Melchionna.

using billions of cells and hundreds of millions biological bodies, say stylized red cells, on leading edge parallel computers (see fig. 4) [38]. The marriage of LB with IBM and perhaps other types of smoothed particle techniques is likely to prove very fruitful for many soft-matter flow applications in the years to come [39].

Post-Newtonian LBs. – LB schemes are not necessarily tied down to Newtonian fluid mechanics: they can and have been extended to both the two major pillars of post-Newtonian physics: quantum mechanics and relativity.

Quantum mechanics. LB for quantum mechanics (QLB) has been developed as early as 1993 [40] but only recently extended to multi-dimensional cases [41]. Essentially, QLB starts from the observation that the Dirac equation can be *literally* cast in the form of a LB equation for a complex pair of up-down movers in the so-called chiral representation, in which the discrete velocities are identified with the discrete spin states. In $d = 1$ the analogy is exact. In $d > 1$ it is not, because spin and momentum can be kept aligned only along one direction, as reflected by the fact that the Dirac streaming matrices cannot be simultaneously diagonalized. However, judicious resort to operator splitting, where propagation in d dimensions is split into a sequence of d one-dimensional streamings, was recently shown to save the day [42].

It is fair to say that quantum LB has not generated any mainstream to date, even though the situation may well change for the future, especially in connection with the exciting prospects of realizing quantum computers on optical lattices and ion traps [43,44].

Relativistic hydrodynamics. More recently, LB has also been extended to the case of relativistic fluids, with special emphasis on quark-gluon plasmas and electron flows in graphene (see fig. 5) [45–47].

Relativistic LB schemes borrow much of the non-relativistic machinery, however with a number of crucial twists. First, unlike their non-relativistic counterpart, relativistic equilibria, known as Maxwell-Juettner (MJ) distribution are *not* separable along the three momentum

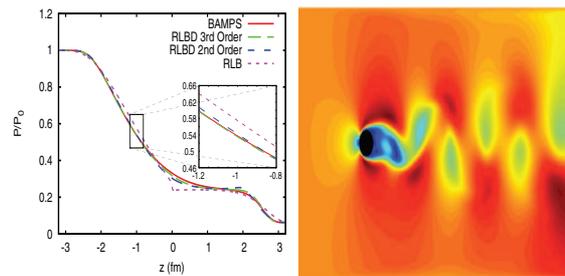


Fig. 5: (Colour on-line) RLB simulations of shock wave propagation in quark-gluon plasmas (left) and electronic pre-turbulence in graphene (right). The left panel shows the pressure distribution computed with different RLB schemes as compared to a full Boltzmann Approach for Multi-Parton Systems (BAMPS). The right panel shows vortex shedding past an impurity obstacle in a graphene sample at Reynolds number of about 100. From [47] and [46].

(velocity) directions. We remind that separability was the guiding light for Maxwell’s derivation of his eponymous distribution, well before Boltzmann came to it as a zero-point of his collision operator.

This is a consequence of the non-separable form of the relativistic energy-momentum relation:

$$E(p) = \sqrt{m^2 c^4 + p^2 c^2}. \quad (5)$$

Unseparability generates a number of technical issues, primarily the lack of simple basis functions like the Hermite polynomials, which have proven very useful in the development of classical LB [48]. Fortunately, this can be remedied by building suitable basis function, using the MJ distribution $f_J(p) \sim e^{-E(p)/k_B T}$ as a weight function.

The second issue regards the inclusion of dissipative effects. As is well known, relativistic dissipative phenomena cannot be described by Laplace operators, for this clashes with the causality requirement of finite-speed propagation. The basic appeal of LB is built-in causality, since LB comes naturally in covariant form, first order in space and time [45]. Nevertheless, recovering the correct relativistic symmetries for dissipative operators requires high-order lattices, matching moments of at least order six. Relativistic LB simulations of shock wave propagation in quark-gluon plasmas have shown excellent performance as compared to full Boltzmann solvers and also *vs.* relativistic hydrodynamic codes [45]. Major work is required to consolidate these findings in connection with three-dimensional simulations of quark-gluon plasmas as they originate, for instance, in relativistic heavy-ion collider experiments.

It is hard to predict whether such post-Newtonian offsprings will ever burst out in a way comparable to the case of classical fluids. Hopefully, major progress will result by a proper mixing of the fluid and high-energy communities.

Holographic fluids. Strongly interacting fluids, such as quark-gluon plasmas, Bose-Einstein condensates in

optical lattices and electron flows in graphene, have gained tremendous attention in the recent years, mainly under the impulse of major experimental breakthroughs in condensed matter and high-energy physics. They also unveiled profound and fascinating connections between fluid mechanics, condensed matter, high-energy physics and even string theory [49,50]. These fluids appear to realize the closest known approximation to a *perfect fluid*, in that they minimize the famous AdS-CFT (Anti-de Sitter, Conformal Field Theory) viscosity bound [51]:

$$B \equiv 4\pi \frac{\eta k_B}{s \hbar} \geq 1, \quad (6)$$

where η is the dynamic viscosity and s the entropy per unit volume. This inequality was derived from the celebrated duality between $(d + 1)$ dimensional gravity and conformal field theory in dimension d . Since they live on the boundary (“brane” in stringy parlance) of the gravitational domain, these fluids are called “holographic” [52]. Although not rigorous, the above inequality appears to be fulfilled by all known fluids, with values of B ranging from 400 for water, to about 10 for helium-3, down to about 1 for quark-gluon plasmas.

At a first glance, holographic fluids stand like the graveyard of kinetic theory. Indeed, it is argued that, due to the very strong interactions, not only Boltzmann kinetic theory, but the very notion of quasi-particle as a weakly interacting collective degree of freedom, may lose significance for holographic fluids, essentially because quasi-particles live too short to carry any sensible information. Interestingly, though, hydrodynamics continues to hold for holographic fluids as disparate as super-hot and dense quark-gluon plasmas and super-cold dilute atomic gases, which lie some thirty orders of magnitude apart in density and even more so in temperature.

Does this “holo-hydrodynamics without kinetic theory” picture spoil the possibility of using lattice kinetic methods? In our opinion, the opposite is true.

To begin with, LB, especially in its entropic version, routinely operates with ephemeral quasi-particles which may live as short as one millionth of their natural free-flight time. This is a gift of the negative propagation viscosity discussed earlier on in this Perspective. More generally, holo-hydrodynamics only adds to the top-down approach described in the early part of this Perspective, namely *designing* LB schemes based on the symmetries of the macroscopic (field theory) target, rather than *deriving* them *ab initio* from microscopic dynamics. Impressive QLB simulations of quantum turbulence in Bose-Einstein condensates already exist [53] and there are good reasons to believe that LB formulations in curved manifolds should be able to simulate efficiently holographic flows such as those reported in fig. 6.

LB as a kinetic generator of non-linear field theories. – Finally, I wish to bring up a rather abstract but general perspective, namely LB as a *kinetic genera-*

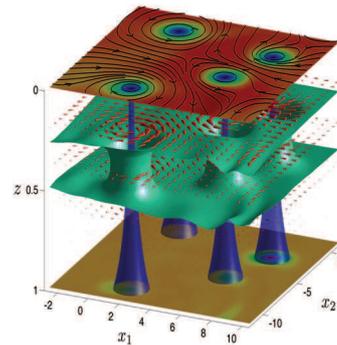


Fig. 6: (Colour on-line) Holographic turbulence (from [52]). The lower plane represents the black-hole horizon and the upper one the boundary of AdS gravity. The bulk is filled with a gauge field, whose condensate eventually prevents excitations from “falling down” in the black-hole horizon. However, due to the formation of vorticity spirals punching vortices in the upper plane, where the superfluid lives, some excitations can fall through the black-holes. This is the way the holographic superfluid eventually dissipates its energy.

tor of Partial Differential Equations (PDEs) describing a broad family of (non-linear) field theories, not only hydrodynamics. The inspiring principle is quite general: kinetic theory is an uplift of continuum field theory (Navier-Stokes) from four-dimensional spacetime to seven-dimensional phase-spacetime. Spacetime-dependent fields are recovered upon projecting the $(6+1)$ -dimensional kinetic equation back to $(3+1)$ -dimensional spacetime by taking kinetic moments, *i.e.* weighted integrals in momentum space. The specific nature of the resulting PDEs depends on the details of the four ingredients described in the initial part of this paper. By using this “kinetic-uplift” principle, several well-known paradigmatic PDEs, say Gross-Pitaevski, Korteweg-de Vries, Kardar-Parisi-Zhang ones, have been cast in LB format. Most papers in this stream show that the LB formulation does reproduce the expected physics. This is conceptually nice, but in the long term it will be important to show where the computational benefits really are. Perhaps the most interesting question is whether the kinetic formulation subtends a real physical picture, namely the existence of physical quasi-particles for the given non-linear field theory in point. This line of research stands as a very broad and potentially far-reaching playground for future research.

Conclusions. – In the short-term, it is rather unriskey to predict that LB will keep growing as a useful tool in all major fields of classical fluid dynamics and soft-matter research: flows in porous media, possibly with allied chemical reactions, turbulence modeling, multiphase and colloidal flows, will continue to take the lion’s share of LB research. Even in such “consolidated” fields, there will always be a steady need for improvement, typically better numerical stability at high-Reynolds numbers and for flows with large density jumps at the interfaces and

significant heat transfer. A similar statement goes for LB versions at the micro- and nanoscale, where it is extremely important to handle fluids with large fluctuations and robust coupling to floating bodies, such as large biomolecules. These developments tend to be highly technical and sometimes even obscured by a rather inextricable tangle of physical and numerical complexities. Even though such complexities might never see a full end, steady incremental progress will surely make a huge difference from here to 2038, with many potential breakthroughs along the way.

As to quantum and relativistic fluids, I sense there might be a goldmine awaiting for fundamental progress. Among others, the idea of using LB methods in the apparently forbidden territory of holographic fluids appears very tantalizing. When looking at Liboff's book [54], I cannot help but think that we have (partially) achieved only the first third of the story, the remaining two being hopefully going to take a substantial share of lattice Boltzmann 2038. If not for anything else, I hope that this Perspective has served the purpose of highlighting the breadth of the LB contribution to the general philosophy of *computing as a tool for discovery*. More importantly, I hope it will stimulate young readers of any age to take on some of the challenges portrayed in this work, with my best wishes to help shaping up a great LB 2038!

* * *

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